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Designing hypothetical learning trajectory based on van hiele theory: a case of geometry

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Designing hypothetical learning trajectory based on van hiele theory: a case of geometry

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Abstract. This study is aimed to develop a hypothesis of learning trajectory based on the van Hieletheory which can assist students in comprehending the volume of cuboid. This study used design research to create a hypothetical learning trajectory for students to learn the volume of cuboid. Out of the three stages of design research, this study focused on the first stage, i.e. preliminary design because the main focus was to develop a sequence of learning activities to learn the volume of cuboid on the basis of van Hieletheory. The results of this study is a HLT on discovering the concept of volume of cuboid. This HLT consists of four stages of van Hiele, i.e. 1) visualization stage at which students discover the volume of cuboid by counting a number of unit cubes construct the solid; 2) analysis phase at which students identified the volume of cuboid by looking for the length of edges without counting unit cubes; 3) informal deduction stage at which students find out the volume of cuboid by looking at the length of the sides, 4) deduction phase at which the students discovered the general formula of volume for all types of cube, and cuboid.

1. Introduction

Planning is required in the process of teaching and learning in the classroom designed in accordance with the needs of teachers and students. It is essentially a short-term designing to estimate or project what will be done in learning [1]. The preparation of a lesson plan needs to take into account the individual differences of learners by gender, initial ability, intellectual level, interest, motivation, talent, potential, social skills, emotions, learning styles, special needs, learning speed, cultural background, norms, and environment of learners [2]. The process of designing the lesson planning needs to be seen from the perspective of students like what and how the learning process will be implemented, in order to align the flow of how the teachers teach and way1s of students in thinking to attain the learning objectives to be achieved. The relationship between teachers, learning materials, students can be illustrated by a didactic triangle, containing the relationship between teacher and students called pedagogical relationships (PR) and the relationship between students with learning materials called didactic relationships (DR) [3].

Pedagogical relationship is used to formulate the knowledge possessed by teachers about mathematics, learning activities and mathematical representation as needed. It aims to create a hypothetical learning trajectory [4]. When the teachers gain insight into students 'thinking and

learning, they will begin to formulate conceptions of students' understanding and use those to develop allegations about the sequence of tasks and learning activities. Thus, they will formulate a hypothetical learning trajectory to map out the instruction [5].

The hypothetical learning trajectory, first proposed by Simon in 1995, reveals that the trajectory of hypothetical learning consists of three components: learning objectives that determine the direction, learning activities, and hypothetical learning process which is the speculation about the development of the students' thinking and understanding in learning activities [4]. The hypothetical learning trajectory clarify the interdependence of activities and learning processes [6]. The trajectory of hypothetical learning is defined by using four principles that are 1) The HLT generation is based on the students' current knowledge understanding; 2) HLT is a container for planning the learning of certain mathematical concepts; 3) the math task provides a tool for promoting the learning of certain mathematical concepts which it is a paramount part of the learning process; 4) Due to the uncertain hypothetical and inherent of this process, teachers are regularly involved in modifying every aspect of HLT [7]. To know more how the HLT design for mathematics learning used, it can be seen by Figure 1.

This study focuses on students' ability to think geometry on cuboid volume material based on van Hiele theory. The volume of cube and cuboid are one of the incumbent topics taught in junior high schools since these are important geometric concepts underlying many aspects of mathematics, such as for learning the volume of polyhedron as well as Curved-face three-dimensional objects [8]. Geometry plays important role in learning other concepts in mathematics learning [9].

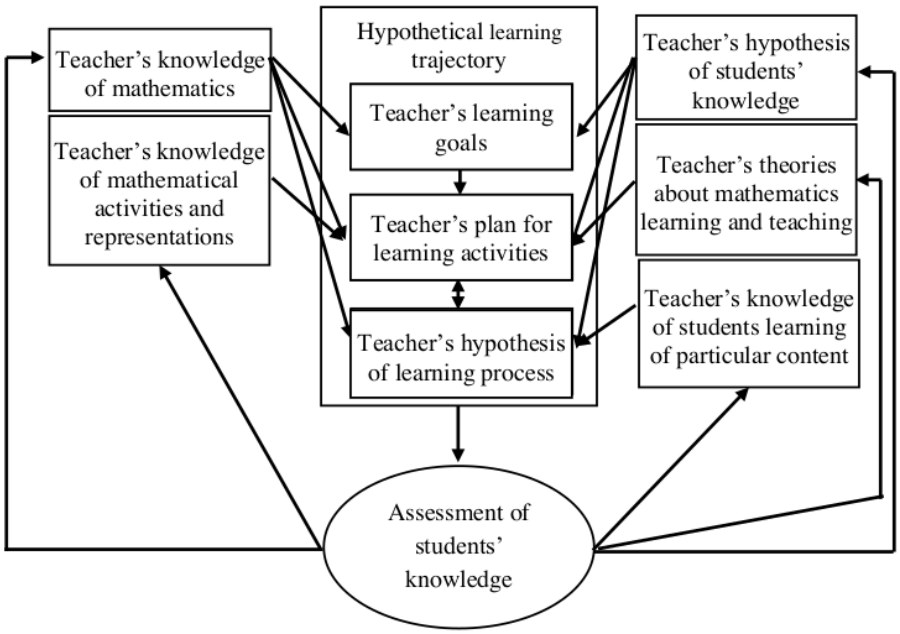

Figure 1. Hypothetical learning trajectory in the learning cycle of mathematics [4].

Based on TIMSS 2011 results from 45 countries in the world, Indonesia ranked 41st for mathematics ability of grade VIII students with average score 386. Specifically the score of Indonesian students at TIMSS 2011 was 377 for geometry domains [10]. Besides, PISA 2015 from 65 countries, Indonesia was in position 64th with average score 386 [11]. To solve the problem with the

characteristics of the PISA and TIMSS questions, students should have the skills as determined in the mathematics learning objectives[12]. Moreover, the average mathematical ability of junior high school students in Indonesia based on the results of national examinations (UN) in the last three years displayed that the average percentage of mathematics examination results of students in 2014/2015 stood at 56.40%. It went down 50.24% in 2015/2016. The next year, it increased to 54.75%.

Percentage of mastery about geometry and mathematical measurement at UN 2016/2017 was 48.57%. According to the UN results, we can conclude that in the learning of geometry and measurement of junior high school and equal in Indonesia have a relatively low percentage because the number of students who answered correctly on the questions were less than 50% of the total junior high school students and equal who participated the UN in Indonesia.

Based on the above description then the question would be "how to anticipate the design of learning trajectory of volume cuboid based on van Hiele theory"

2. Theoretical Background

2.1 Hypothetical Learning Trajectory

Hypothetical learning trajectory(HLT) is as a way to explicate an important aspect of pedagogical thinking involved in teaching mathematics for understanding. In particular, it described how mathematics educators (i.e., teachers, researchers, and curriculum developers), oriented by a constructivist perspective and particular mathematics learning goals for students, can think about the design and use of mathematical tasks to promote mathematical conceptual learning [7]. Defined a conceptual analysis as ways of thinking that, if students had them, might be propitious for building more powerful ways to deal mathematically with their environments than they would build otherwise [13]. Thompson described conceptual analysis as useful in two ways. First, one can generate models of thinking that aid in explaining observed behaviors and actions of students (part of an emergent learning trajectory). Second, one can construct ways of understanding that, were a student to have them, might be useful for his or her development of a scheme of meanings that would constitute a coherent conception of a mathematical idea [13].

This HLT can be analogized by planning a travel route. If we comprehend the possible routes to the destination, we can select a route that is possible to go to our destination then we can opt a good route [14]. By knowing the student's learning trajectory, the teacher can get a proper learning trajectory and can be used to assist students apprehending math. HLT is divided into three parts. The first part of the teacher's learning goals is influenced by two factors: (1) teacher's knowledge of mathematics the importance of teacher's understanding is because the teacher have to know the sequence of the continuous learning flow of mathematics and (2) the teacher's hypothesis of students' knowledge. Teachers also should find out the extent to which students understand the lessons being taught, so that the process of teaching in the classroom goes to maximum. For more details can be seen in the HLT compiled by Simon in figure 1.

Simon used the HLT as part of the so-called Mathematics Teaching Cycle, mostly for one or two lessons, but we utilize it as an instrument in design research for longer sequences of instruction [15]. The HLT is the link between an instruction theory and a concrete teaching experiment. It is informed by general domain-specific and conjectured instruction theories, and it informs researchers and teachers how to carry out a particular teaching experiment [16]. This means that an HLT, after it has been mapped out, has different functions depending on the phase of the design research and continually develops through the different phases. It can even alter during a teaching experiment [15].

- During the design phase, the HLT, once formulated, guides the design of instructional materials that have to be developed or adapted. The confrontation of a general rationale with concrete activities often leads to a more specific HLT, which means that the HLT usually develops during the design phase [17]

- During the teaching experiment, the HLT functions as a guideline for the teacher and researcher what to focus on in teaching, interviewing, and observing. It can happen that the teacher or researcher feels the need to adjust the HLT or instructional activity for the next lesson [15].
- During the retrospective analysis, the HLT functions as a guideline determining what the researcher should focus on in the analysis. Because predictions are made about students' learning, the researcher can contrast those anticipations with the observations made during the teaching experiment [15].

These three steps are seen in figure 2. In the cycle of how to design HLT in a design study used by Gravemeijer and Cobb [18].

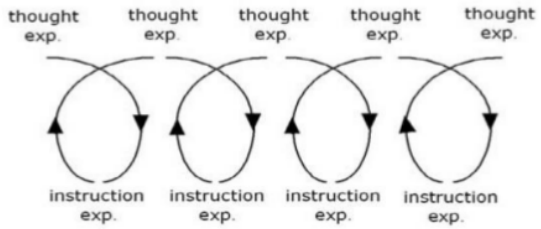

Figure 2. Design HLT by Gravemeijer and Cobb [18].

2.2 Van Hiele Theory

Van Hiele forms a five-tier framework that has been the subject of numerous research worldwide. Van Hiele's theory does not offer a deterministic view of steady progress, but an empirical description of a relatively stable stage that can give educators a guide to constructs learners' experiences properly [19]. Each level in the van Hiele is a definite character of activity and instruction that has implications for teachers [20]. So the geometric development of students depends on the participation of students in the classroom. Therefore, instruction should always follow the patterns of students' thinking behavior and should be aimed to encourage the development from one level to the next. [21]. Stages of geometry thinking based on van Hiele theory can be seen in figure 3, while the fifth stages of thinking level of van Hiele geometry are:

2.2.1 Level 1 (Recognition/Visualization)

In the first stage of the model, students observe the object in gestalt, and decisions are mostly perception based rather than reasoning. And students treat the figure without its traits, definitions and descriptions [20]. A learner at this level identifies, names, compares and operates on geometrical shapes such as triangles, angles and parallel lines according to their appearances [22]. At level 1 a learner can recognize shapes by their appearance but cannot identify specific properties of shapes [19]. The products of thought at level 1 are classes or groupings of shapes that seem to be "alike" [23].

2.2.2 Level 2 (Analysis)

At this level, students identify the traits of the object, figure or shape. In which students cannot define and describe the object completely [20]. A learner analyses the attributes of shapes and the relationships between the attributes of shapes [22]. Murray notes, that at level 2 the terminology and symbols are meaningful to learners and they can formulate their own definitions. Definitions are accepted as binding for logical arguments and discussions [19]. The products of thought at level 2 are the properties of shapes [23].

2.2.3 Level 3 (Informal Deduction)

At this stage, students are capable of reasoning with meaningful description [20]. They can also logically interrelates previously discovered properties/ rules by giving or following informal arguments [22]. The products of thought at level 3 are relationship among properties of geometric objects [23].

2.2.4 Level 4 (Deduction)

At this level, students can construct proofs [20]. The student proves theorems deductively and establishes interrelationships among networks of theorem [22]. The products of thought at level 4 are deductive axiomatic systems for geometry [23].

2.2.5 Level 5 (Rigor)

Students at this level understand the formal aspects of deduction [20]. The student establishes theorems in different postulation systems and analyzes/ compares these system [22]. The products of thought at level 5 are comparisons and contracts among different axiomatic systems for geometry [23].

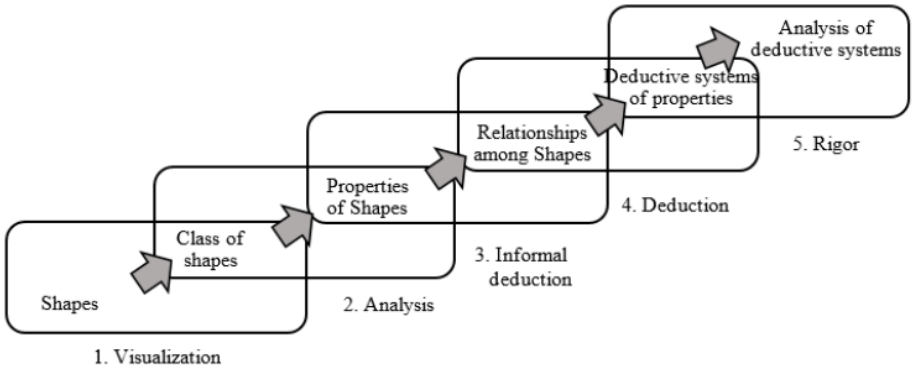

Figure 3. The van Hiele theory of geometric thought [23].

The five levels are interconnected so that to go to the next level students have to complete the previous level. This research focus on visualization, analysis, informal deduction and deduction. The van Hiele developed a framework for organizing classroom instruction to aid teachers' structure activities that cultivate their students' geometric thinking [24]. This framework includes a sequence of five phases of learning: information/inquiry, directed orientation, explication, free orientation, and integration (see Table 1).

Table 1. The van Hiele sequence of phase of learning provides a framework for teachers to guide students through the levels of understanding.

Phase	Description
Information/ inquiry	Teacher: Assess students' prior knowledge through discussion and allow question to prompt topics to be explored
Directed orientation	Teacher and students: Explore sets of carefully sequenced activities
Explication	Students: Share explicit views and understandings about the activities
Free orientation	Teacher: Challenge students to solve problems related to the geometric concepts and make connections among them
Integration	Students: Reflect on observations and how they fit into the overall structure of the concepts

Sources: Adapted from Geddes [25].

3. Methods

This study utilized design research as a research methodology type validation studies to create a learning trajectorydesign framework to be traversed by students in mathematics learning. The stages done in the research design was divided into three namely (1) preparing for the experiment (preliminary design); (2) experimenting in the classroom; and (3) conducting retrospective analyses [18]. We focused on preparing for the experiment (preliminary design) aiming to formulate a local instruction theory that could be elaborated and refined while conducting the experiment.

Design research was a methodology that has five characteristics: interventionist nature; process oriented; a reflective component; a cyclic character; and theory oriented [18]. Design research was a cyclical process of thought experiment and instruction experiments [17]. There are two important aspects related to design research. They are the hypothetical learning trajectory (HLT) and local instruction theory (LIT) [26].

The researcher hypothesize a learning in geometry lesson of junior high school that was adapted to the mastery of the student geometry concept. This study was conducted on the geometry of polyhedron focusing on the volume of cube and cuboid by using four stages of van Hiele theory.

4. Results

This lesson is designed to devise a learning trajectory hypothesis in the learning of cube and cuboid's volume materials using four stages of van Hiele theory. The hypothesis of how to infer the thinking flow of students and actions undertaken by teachers based on learning objectives that refer to the curriculum in Indonesia. The learning objectives of cube and cuboid's volume are: (1) determine the volume of cube and cuboid; and (2) solving problems related to the volume of cube and cuboid [27].

For learning activities and the students' thinking flow hypothesis is conducted according to the four phases of van Hiele theory. In the first stage of the level of visualization, activity and hypothesis can be seen in table 2.

Table 2. Level 1 Visualization.

Activities	Teacher Activity	Hypothesis of Student Thinking Flow
Information	Provides information about gigo connect a cube that builds up cube and cuboid.	Students observed the appearance of the gigo connect a cube
Direct orientation	Teacher encourage students to develop blocks of gigo connect a cube into a cube and cuboid, which the size is exemplified in advance	Students construct blocks of gigo connect a cube into a cube and cuboid, which the size is exemplified previously
Explication	The teacher asks the students to explain how many unit of blocks form cube and cuboid	Students explain the formation of cubes and cuboids with a certain size of the unit of the blocks
Free orientation	The teacher asks the students to detect the desired volume of cubes and cuboids by counting the number of unit blocks formed	Students search for the desired volume of cubes and cuboids by counting the number of unit blocks formed
Integration	The teacher asks the students to conclude what is obtained at this stage	Students conclude that the cubes and cuboids of a certain size, formed from blocks of gigo connect a cube whereas the number depends on the size of the model.

At this stage, students can comprehend and deduce the volume of a cube and cuboid by counting the small cubes or blocks of gigo connect of forming the polyhedron. This understanding occurs when the students are visibly exposed the exists polyhedron so that the calculations are based on

observations conducted by the students. After given the real appearance then the next step given by the students is the analysis (see table 3).

Table 3. Level 2 Analysis.

Activities	Teacher Activity	Hypothesis of Student Thinking Flow
Information	The teacher gives information about cubes and cuboid in the form of 2-dimensional images that remain to have unit cubes	Students observe the information provided.
Direct orientation	The teacher asks the students to determine the lengths of the sides of the cubes and cuboid formed on the unit cubes	Students determine the lengths of the sides of the cubes and cuboid formed on the unit cubes
Explication	The teacher asks the students to explain what they observed	Students explain that the cuboid are composed of p unit length, l unit width, and t unit height is formed by unit cubes that construct it.
Free orientation	The teacher invites students to determine the size of the cube and other cuboid and how many unit cubes are obtained when they have such size	Students determine the size of the cube and other cuboid and how many unit cubes are obtained when they have such size
Integration	The teacher asks the students to conclude what is obtained at this stage	Students concluded that to find the volume of the cube and cuboid, firstly the student have to know the lengths of the sides of the cube and the cuboid and then multiply it.

At this phase, it is seen that to be able to determine the volume of cubes and cuboid, students remain to count the number of unit cubes possessed by cubes and cuboid to be searched, but the students do it, not to count one by one but just multiplied by the constituent elements. After the stages of analysis can be understood then the students can go to the level of informal deduction, activity and hypothesis can be seen in table 4.

Table 4. Level 3 Informal Deduction.

Activities	Teacher Activity	Hypothesis of Student Thinking Flow
Information	The teacher displays the picture of cube and cuboid that is known only to the lengths of its sides	Students observe what is delivered
Direct orientation	The teacher asks the students to determine the edge of the cube and the cuboid.	Students determine the edge of the cube and the cuboid.
Explication	Teachers ask students to explain in advance what steps need to be done	The student explains the steps performed to obtain the cube and cuboid volume formula.
Free orientation	The teacher asks students to look for the formula of the cube and cuboid volume	Students look for the formula of the cube and cuboid volume
Integration	The teacher asks the students to conclude what is obtained at this stage	Students conclude that the cube and cuboid volume formula is $V = p \times l \times t$

At this phase, students have been able to determine the volume of cubes and cuboids, so that the students are easy to determine the volume of cubes and cuboids simply by knowing the length of the edges. After this stage is complete then the students can go to the level of deduction, for the activity and learning hypothesis can be seen in table 5.

Table 5. Level 4 Deduction.

Activities	Teacher Activity	Hypothesis of Student Thinking Flow
Information	The teacher gives information about the volume of cubes and cuboids	Students observe the information provided
Direct orientation	The teacher asks students to prove the volume of cubes and cuboids by using cube and cuboid images then students will divide the length of cube and cuboid as much as a unit cube with the size of 1 unit.	Students prove the volume of cubes and cuboids by utilizing the images of cubes and cuboids then students will separate the length of the cube and cuboid as much as a unit cube with the size of 1 unit.
Explication	The teacher asks the students to give their views	Students give their views on what is given
Free orientation	The teacher asks the students to prove the problem	Students complete and prove what is the main problem
Integration	The teacher asks the students to conclude what is obtained at this stage	Students concluded correctly that the cube and cuboid volume formulas can be used for different sizes of cubes and cuboids.

At this stage, students do a mathematical proof of the volume of cubes and cuboids obtained, so that at this level the mathematical concepts of students are tested to prove the geometry problems possessed by the students. These four stages are a series of paths that must be passed by the students to be able to understand the concept of the volume of cubes and cuboids. Step by step is very important because it can be used as a benchmark for further learning.

The following scheme illustrates an example of a series of geometry learning activities based on van Hiele theory hypothesized according to students' thinking flow (see figure 4)

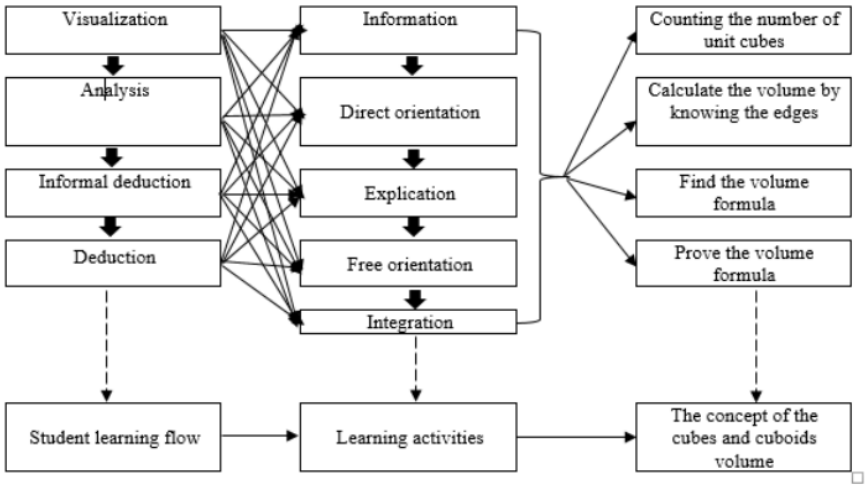

Figure 4. The hypothesis of students' thinking flow in the learning of cube and cuboid volume.

5. Conclusion

Based on the description of the example of hypothetical learning trajectory implementation, it can be concluded that hypothetical learning trajectory in the volume of cubes and cuboids can be used as instructional instructions for teachers to achieve the desired learning objectives. This knowledge represents the main purpose of mathematics in the development of learning [28]. Hypothetical learning

trajectory implementation,it can be useful as a guide to the implementation of learning as well as to provide alternative strategies to overcome difficulties in understanding the concept of the material being studied.These approaches had been tested and it gave positive results; therefore they thought their method as an effective method to teach mathematics [5].

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